Finite-size effects in the superconformal β-deformed $\mathcal{N} = 4$ SYM

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ABSTRACT: We study finite size effects for composite operators in the SU(2) sector of the superconformal β-deformed $\mathcal{N} = 4$ SYM theory. In particular we concentrate on the spectrum of one single magnon. Since in this theory one-impurity states are non BPS we compute their anomalous dimensions including wrapping contributions up to four loops and discuss higher order effects.

Contents

1. Introduction

Recently remarkable progress has been achieved in studying integrability properties within the arena of the AdS/CFT correspondence [1]. The ideal setting for performing this analysis is provided by the planar limit of the superconformal $\mathcal{N} = 4$ SYM theory and its string dual in the $\mathrm{AdS}_5 \times \mathrm{S}^5$ background. Indeed major efforts have been devoted to the comparison of the spectra on the two sides of the correspondence, i.e. the anomalous dimensions of gauge invariant operators in the gauge theory versus the mass spectra in the corresponding string sector. Higher loop perturbative calculations in the gauge theory have been performed taking advantage of the quantum spin chain description for which an Hamiltonian and a corresponding asymptotic Bethe ansatz can be constructed [2–6]. In particular important results have been obtained for operators of infinite length, since in this case the dynamics simplifies considerably and it is essentially encoded in an exact, factorized S-matrix corrected by a dressing phase [7–10].

In order to deepen our understanding it becomes crucial to take into account finite size effects. On the string theory side there have been recent papers addressing this issue and studying finite size contributions in the spectrum of magnons [11–18]. In [19] wrapping effects in some toy models were studied.

On the field theory side, i.e. in the $\mathcal{N} = 4$ SYM theory, finite size effects arise from wrapping interactions [6, 20]. The simplest situation in which this kind of interactions shows up is at four loops in the anomalous dimensions of the composite operator $\mathrm{tr}(\phi[Z, \phi]Z)$. We have performed this calculation [21, 22] and found a new type of contributions proportional to $\zeta(5)$ that increases the order of transcendentality of the anomalous

dimension.[1] While this result is not in contradiction with the Kotikov-Lipatov transcendentality in the universal scaling function, it was not expected in previous conjectures for the anomalous dimension of the composite operator [24, 10, 25]. It would be nice to have an independent check of our result,[2] but if one attempts higher order calculations the algebra becomes immediately too cumbersome to deal with. Alternatively one can study a less symmetric theory that might exhibit new features to be compared to string theory known results. Such an example is provided by an exactly marginal deformation of $\mathcal{N} = 4$ SYM theory preserving $\mathcal{N} = 1$ supersymmetry. The deformed theory is obtained modifying the original $\mathcal{N} = 4$ superpotential for the chiral superfields in the following way

$$ig\,\mathrm{tr}\,(\phi\,\psi\,Z - \phi\,Z\,\psi) \;\longrightarrow\; ih\,\mathrm{tr}\left(\mathrm{e}^{i\pi\beta}\,\phi\,\psi\,Z - \mathrm{e}^{-i\pi\beta}\,\phi\,Z\,\psi\right) \tag{1.1}$$

where in general h and β are complex constants. In [27] it was argued that this β-deformed $\mathcal{N} = 1$ theory becomes conformally invariant, i.e. the deformation becomes exactly marginal, if one condition is satisfied by the constants h and β. More precisely it has been shown that for a real deformation parameter this theory becomes superconformal, in the planar limit to all perturbative orders [28], if

$$h\bar{h} = g^2\,, \tag{1.2}$$

where $g = g_{\mathrm{YM}}$ is the Yang-Mills coupling constant. We also define the 't Hooft coupling constant

$$\lambda = \frac{g^2 N}{16\pi^2}\,. \tag{1.3}$$

Via the AdS/CFT correspondence this β-deformed $\mathcal{N} = 4$ SYM theory is expected to be equivalent to the Lunin-Maldacena string theory background [29]. The existence of integrable structures in the deformed string background has been analyzed in [30, 31]. Finite size effects of single magnons were discussed in [32]. Integrability of the deformed field theory was studied in [33, 34].

In this paper we want to study the anomalous dimension of short operators in the superconformal deformed $\mathcal{N} = 4$ SYM theory including wrapping contributions to be compared to finite size effects in the corresponding string theory.

The anomalous dimension for a composite operator $\mathcal{O}$ is extracted from the $1/\varepsilon$ pole of the graphs contributing to its renormalization: for an operator undergoing multiplicative renormalization it is defined as

$$\gamma(\mathcal{O}) = \lim_{\varepsilon\to 0}\left[\varepsilon g\frac{\mathrm{d}}{\mathrm{d}g}\log\mathcal{Z}_{\mathcal{O}}(g,\varepsilon)\right]\,, \tag{1.4}$$

where

$$\mathcal{O}_{\mathrm{ren}} = \mathcal{Z}_{\mathcal{O}}\mathcal{O}_{\mathrm{bare}}\,.$$

In presence of mixing among different operators, the second equation should be understood in matrix form and the first one is still valid for the eigenstates of the renormalization matrix.

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[1]Following our paper a four-loop calculation, not in accordance with our result, was presented in [23].

[2]Such a check is provided by the later result in [26].

Our strategy will be the following:

We perform perturbative calculations using a superfield approach. All the superspace conventions, D-algebra techniques and shortcuts are explained in detail in [35, 22] and will not be repeated here.

We compute higher-loop integrals using the Gegenbauer polynomial x-space technique [36] which we will review and refine in a forthcoming publication [37]. We will also make use repeatedly of the results already listed in [22].

Here we focus primarily on the calculations of the anomalous dimensions of composite operators that exhibit relevant differences as compared to the $\mathcal{N} = 4$ SYM case. The major novelty of the deformed theory is given by the fact that one-impurity states are not protected by supersymmetry. The shortest[3] such a state is given by the length-three, single-impurity operator

$$\mathcal{O}_{1,3} = \text{tr}(\phi ZZ) \ . \tag{1.5}$$

We will compute its anomalous dimensions up to three loops. In order to achieve this goal we proceed following the same lines of reasoning as in [21, 22].

First we write the dilatation operator up to the third order for the β-deformed theory. In this way we obtain the correct Hamiltonian only for operators in the asymptotic limit.

In order to derive the correct result when it is applied to a state of length three, we have to subtract the range four interactions and add explicitly the wrapping contributions. This is done in sections 2 and 3. (More precisely in section 2 we compute the deformed dilatation operator up to fourth order since it will be useful later on).

Then using the same technique we compute the anomalous dimension of the length-four, single state operator

$$\mathcal{O}_{1,4} = \text{tr}(\phi ZZZ) \tag{1.6}$$

up to four loops. This computation is presented in the first part of section 4.

We turn to the length-four, two-impurity operators

$$\text{tr}(\phi\phi ZZ) , \qquad \text{tr}(\phi Z\phi Z) \tag{1.7}$$

in the second part of section 4, where we exploit the knowledge of the wrapping dilatation operator for the undeformed case [22] to compute their anomalous dimensions.

Finally we analyze again the simplest single-impurity operator $\mathcal{O}_{1,L} = \text{tr}(\phi Z^{L-1})$, and we attempt to go to higher order L in perturbation theory. Even if exact calculations are too difficult and out of reach, still we are able to compute whole classes of diagrams that allow us to make some plausible conjectures. These are described and collected in the last section of the paper.

2. Dilatation operator

As anticipated in the introduction in order to compute the anomalous dimensions of single-impurity operators it is convenient to make use of the asymptotic dilatation operator. In

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[3]The length-two operator $\text{tr}(\phi Z)$ was shown to be protected in [38, 39].

fact this allows to avoid the explicit computation of a large number of diagrams. Thus we need derive the expression of the asymptotic dilatation operator for the β-deformed theory. We now show how this can be obtained from the knowledge of the Hamiltonian of the $\mathcal{N} = 4$ theory.

First of all we recall the form of a standard permutation of fields at sites i and j which is given by

$$\mathrm{P}_{i,j} = \frac{1}{2} \left[\mathbb{1}_{i,j} + \vec{\sigma}_i \cdot \vec{\sigma}_j \right] = \frac{1}{2} \left[\mathbb{1}_{i,j} + \sigma_i^3 \sigma_j^3 + \sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+ \right] . \tag{2.1}$$

Using these permutations we can build a set of standard basis operators

$$\{a_1, \ldots, a_n\} = \sum_{r=0}^{L-1} \mathrm{P}_{a_1+r, \, a_1+r+1} \cdots \mathrm{P}_{a_n+r, \, a_n+r+1} . \tag{2.2}$$

The dilatation operator for the $\mathcal{N} = 4$ theory can be written in terms of these operators.

We now look for a similar set of basis operators for the deformed theory. To this end we consider deformed permutations [33]

$$\mathbf{P}_{i,j} = \frac{1}{2} \left[\mathbb{1}_{i,j} + \sigma_i^3 \sigma_j^3 + q^2 \, \sigma_i^+ \sigma_j^- + \bar{q}^2 \, \sigma_i^- \sigma_j^+ \right] , \qquad q \equiv \mathrm{e}^{i\pi\beta} \tag{2.3}$$

and define corresponding deformed basis operators:

$$\{a_1, \ldots, a_n\} = \sum_{r=0}^{L-1} \mathbf{P}_{a_1+r, \, a_1+r+1} \cdots \mathbf{P}_{a_n+r, \, a_n+r+1} . \tag{2.4}$$

Using these definitions the expansions of the chiral structures of Feynman diagrams in terms of basis operators have exactly the same coefficients as in the undeformed theory [22]. They are given by

$$\begin{aligned}
\chi(a,b,c,d) &= \{\} - 4\{1\} + \{a,b\} + \{a,c\} + \{a,d\} + \{b,c\} + \{b,d\} + \{c,d\} \\
&\quad - \{a,b,c\} - \{a,b,d\} - \{a,c,d\} - \{b,c,d\} + \{a,b,c,d\} , \\
\chi(a,b,c) &= -\{\} + 3\{1\} - \{a,b\} - \{a,c\} - \{b,c\} + \{a,b,c\} , \\
\chi(a,b) &= \{\} - 2\{1\} + \{a,b\} , \\
\chi(1) &= -\{\} + \{1\} , \\
\chi() &= \{\} .
\end{aligned} \tag{2.5}$$

Clearly since

$$\lim_{q, \bar{q} \to 1} \{a, b, \ldots \} = \{a, b, \ldots \} \tag{2.6}$$

all our deformed expressions reduce to the correct $\mathcal{N} = 4$ expressions in the limit $q, \bar{q} \to 1$. Therefore the dependence on q and $\bar{q}$ is encoded entirely in the deformed basis operators (2.4) and we can look for the dilatation operator as a linear combination of operators (2.4) with coefficients which are independent of $q, \bar{q}$.

Since in the limit $q, \bar{q} \to 1$ the deformed theory reduces correctly to the $\mathcal{N} = 4$ theory, the above observations allow us to conclude that the asymptotic dilatation operator of the

$$\boxed{\begin{aligned}
\boldsymbol{D_0} &= \boldsymbol{\chi}() \\
\boldsymbol{D_1} &= -2\boldsymbol{\chi}(1) \\
\boldsymbol{D_2} &= 4\boldsymbol{\chi}(1) - 2[\boldsymbol{\chi}(1,2) + \boldsymbol{\chi}(2,1)] \\
\boldsymbol{D_3} &= -24\boldsymbol{\chi}(1) + 16[\boldsymbol{\chi}(1,2) + \boldsymbol{\chi}(2,1)] - 4\boldsymbol{\chi}(1,3) \\
&\quad - 4i\epsilon_2\boldsymbol{\chi}(1,3,2) + 4i\epsilon_2\boldsymbol{\chi}(2,1,3) - 4[\boldsymbol{\chi}(1,2,3) + \boldsymbol{\chi}(3,2,1)] \\
\boldsymbol{D_4} &= +200\boldsymbol{\chi}(1) - 150[\boldsymbol{\chi}(1,2) + \boldsymbol{\chi}(2,1)] + 8(10 + \epsilon_{3a})\boldsymbol{\chi}(1,3) - 4\boldsymbol{\chi}(1,4) \\
&\quad + 60[\boldsymbol{\chi}(1,2,3) + \boldsymbol{\chi}(3,2,1)] \\
&\quad + (8 + 8\zeta(3) + 4\epsilon_{3a} - 4i\epsilon_{3b} + 2i\epsilon_{3c} - 4i\epsilon_{3d})\boldsymbol{\chi}(1,3,2) \\
&\quad + (8 + 8\zeta(3) + 4\epsilon_{3a} + 4i\epsilon_{3b} - 2i\epsilon_{3c} + 4i\epsilon_{3d})\boldsymbol{\chi}(2,1,3) \\
&\quad - (4 + 4i\epsilon_{3b} + 2i\epsilon_{3c})[\boldsymbol{\chi}(1,2,4) + \boldsymbol{\chi}(1,4,3)] \\
&\quad - (4 - 4i\epsilon_{3b} - 2i\epsilon_{3c})[\boldsymbol{\chi}(1,3,4) + \boldsymbol{\chi}(2,1,4)] \\
&\quad - (12 + 8\zeta(3) + 4\epsilon_{3a})\boldsymbol{\chi}(2,1,3,2) \\
&\quad + (18 + 4\epsilon_{3a})[\boldsymbol{\chi}(1,3,2,4) + \boldsymbol{\chi}(2,1,4,3)] \\
&\quad - (8 + 2\epsilon_{3a} + 2i\epsilon_{3b})[\boldsymbol{\chi}(1,2,4,3) + \boldsymbol{\chi}(1,4,3,2)] \\
&\quad - (8 + 2\epsilon_{3a} - 2i\epsilon_{3b})[\boldsymbol{\chi}(2,1,3,4) + \boldsymbol{\chi}(3,2,1,4)] \\
&\quad - 10[\boldsymbol{\chi}(1,2,3,4) + \boldsymbol{\chi}(4,3,2,1)]
\end{aligned}}$$

Table 1: Dilatation operator for the β-deformed theory up to four loops.

deformed theory is simply given by the corresponding dilatation operator of the $\mathcal{N} = 4$ SYM theory [40] through the substitutions

$$\{a_1, \ldots, a_n\} \rightarrow \{a_1, \ldots, a_n\} . \tag{2.7}$$

The deformed dilatation operator up to four loops is given explicitly in table 1. We have verified that its eigenvalues agree with the solutions of the deformed Bethe equations [34].

The knowledge of the asymptotic dilatation operator is very useful since it allows to compute the anomalous dimensions of long composite operators, more precisely operators with a length such that wrapping interactions do not contribute. As emphasized above, in the β-deformed theory single-impurity states of the SU(2) sector are not protected in general. If the corresponding operator $\mathcal{O}_{\mathrm{as}}$ is long enough to avoid wrapping interactions, the anomalous dimension for such a state at a given perturbative order can be obtained from the dilatation operator or alternatively from the all-loop result [28]

$$\gamma(\mathcal{O}_{\mathrm{as}}) = -1 + \sqrt{1 + g^2\left|q - \frac{1}{q}\right|^2 \frac{N}{4\pi^2}} = -1 + \sqrt{1 + 4g^2\sin^2(\pi\beta)\frac{N}{4\pi^2}} . \tag{2.8}$$

In the next two sections we will use these results for the computations of the anomalous dimensions of short one-impurity states where finite size effects become important.
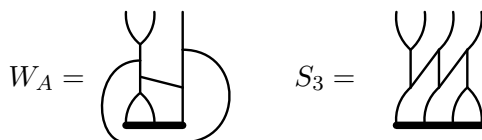
Figure 1: Completely chiral diagrams (wrapping and subtraction).

3. One-impurity state at three loops

The three-loop contribution to the anomalous dimension of any asymptotic (i.e. of length greater than or equal to four) single-impurity operator $\mathcal{O}_{\mathrm{as}}$ is given by

$$\gamma_3(\mathcal{O}_{\mathrm{as}}) = 256\,\lambda^3 \sin^6(\pi\beta) \; . \tag{3.1}$$

This is not a priori the correct value for the anomalous dimension of the length-three, single-impurity operator $\mathcal{O}_{1,3} = \mathrm{tr}(\phi ZZ)$ which is the shortest non-protected operator in the SU(2) sector.

Its exact anomalous dimension at three loops, taking wrapping interactions into account, can be obtained from the result for long states (3.1) in two steps:

- subtract from (3.1) the contributions of range four diagrams, which are not allowed for the length-three operator,

- add the contributions of wrapping diagrams.

As explained in [22], range four diagrams where one line is connected to the rest of the diagram by a single vector line sum up to zero. Therefore the only relevant range four, single-impurity diagram is the one denoted by S_3 in figure 1. Its contribution is given by

$$S_3 \to (g^2 N)^3\,(q-\bar{q})^2\,(q^4+\bar{q}^4)\,I_0 \sim -\frac{16}{3}\frac{\lambda^3}{\varepsilon}\sin^2(\pi\beta)\cos(4\pi\beta)\,, \tag{3.2}$$

where I_0 is the momentum integral shown in table 3, the arrow denotes the result after D-algebra and the $\sim$ symbol means that we have kept only the $1/\varepsilon$ pole contribution. The corresponding term we have to subtract from the asymptotic value (3.1) is

$$\delta\gamma_3^{\mathrm{s}} = -6\lim_{\varepsilon\to 0}(\varepsilon S_3) = 32\lambda^3 \sin^2(\pi\beta)\cos(4\pi\beta)\; . \tag{3.3}$$

Now we consider wrapping contributions. The various diagrams can be grouped into three classes according to their chiral structure. There is one single diagram with only chiral lines, the one labeled W_A in figure 1. It gives

$$W_A \to (g^2 N)^3\,(q-\bar{q})^2\,(q^4+\bar{q}^4)\,I_0 \sim -\frac{16}{3}\frac{\lambda^3}{\varepsilon}\sin^2(\pi\beta)\cos(4\pi\beta)\; . \tag{3.4}$$

Wrapping diagrams with chiral structures $\boldsymbol{\chi}(2,1)$ and $\boldsymbol{\chi}(1)$ are shown in the appendix, in figures 3 and 4 respectively. Graphs with chiral structure $\boldsymbol{\chi}(1,2)$, which is the reflection of $\boldsymbol{\chi}(2,1)$, contribute exactly in the same manner. After D-algebra we find that the diagrams

in each class sum up to zero separately. Therefore these chiral structures do not contribute to the anomalous dimension. In the tables 4 and 5 we show a possible way to combine these diagrams in pairs to have a full, explicit cancellation.

We conclude that the only wrapping contribution to the anomalous dimension of $\mathcal{O}_{1,3}$ comes from W_A and it is equal to

$$\delta\gamma_3^{\mathrm{w}} = -6 \lim_{\varepsilon\to0}(\varepsilon W_A) = 32\lambda^3 \sin^2(\pi\beta)\cos(4\pi\beta) \ . \tag{3.5}$$

The correct three-loop anomalous dimension of $\mathcal{O}_{1,3}$ is then given by

$$\gamma_3(\mathcal{O}_{1,3}) = \gamma_3(\mathcal{O}_{\mathrm{as}}) - \delta\gamma_3^{\mathrm{s}} + \delta\gamma_3^{\mathrm{w}} = 256\,\lambda^3 \sin^6(\pi\beta) \ . \tag{3.6}$$

Since $\delta\gamma_3^{\mathrm{s}} = \delta\gamma_3^{\mathrm{w}}$ the final result happens to be equal to the asymptotic one in (3.1). This cancellation is very likely to be a peculiarity of the three loop calculation and we expect it would no longer hold at four loops. In fact as we will see in the next section the four-loop anomalous dimension of the length-four $\mathcal{O}_{1,4}$ operator is different from its asymptotic value.

It would be nice to perform the four-loop calculation for $\mathcal{O}_{1,3}$ but at the fourth order one has to deal with the insurgence of the three-vector vertex which increases dramatically the number and the complexity of the relevant diagrams to be computed.

4. Four loops

In this section we compute the anomalous dimension of one- and two-impurity states at four loops. In [22] we wrote down the dilatation operator comprehensive of wrapping contributions on the length-four sector. This operator can be easily deformed as we have done for the asymptotic case in section 2. It can be used to obtain the anomalous dimensions both for one- and two-impurity states. However, in the single-impurity case, we chose to do an explicit calculation as in the three-loop case, to check our results and to avoid to use the integrability hypothesis which is hidden in the determination of the four-loop asymptotic dilatation operator.[4]

4.1 Single-impurity state

Here we apply the same technique presented in the previous section to compute the exact anomalous dimension of the length-four, single-impurity state $\mathcal{O}_{1,4}$ at four loops. Again we start from the asymptotic result, valid for any single-impurity state $\mathcal{O}_{\mathrm{as}}$ of length greater than four:

$$\gamma_4(\mathcal{O}_{\mathrm{as}}) = -2560\,\lambda^4 \sin^8(\pi\beta) \ . \tag{4.1}$$

As in the previous case one single diagram must be subtracted in order to get rid of the range five contributions. It contains only chiral interactions and it is shown in figure 2.

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[4]We use (2.8) which does not rely on integrability.

$$S_4 = $$
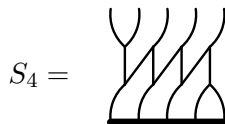
Figure 2: Range-five diagram.

W_{A2}	$\rightarrow$	$(g^2N)^4I_2\left[\boldsymbol{\chi}(1,4,3,2)+\boldsymbol{\chi}(4,1,2,3)\right]$
W_{B1}	$\rightarrow$	$-(g^2N)^4(I_3+I_4+2I_5)\left[\boldsymbol{\chi}(1,2,3)+\boldsymbol{\chi}(3,2,1)\right]$
W_{B2}	$\rightarrow$	$(g^2N)^4I_3\left[\boldsymbol{\chi}(1,2,3)+\boldsymbol{\chi}(3,2,1)\right]$
W_{B3}	$\rightarrow$	$-(g^2N)^4I_2\left[\boldsymbol{\chi}(1,2,3)+\boldsymbol{\chi}(3,2,1)\right]$
W_{B4}	$\rightarrow$	$(g^2N)^4(I_2+I_4+2I_6)\left[\boldsymbol{\chi}(1,2,3)+\boldsymbol{\chi}(3,2,1)\right]$
$\sum W_{B**}$	$\rightarrow$	$-2(g^2N)^4(I_5-I_6)\left[\boldsymbol{\chi}(1,2,3)+\boldsymbol{\chi}(3,2,1)\right]$
W_{E2}	$\rightarrow$	$-(g^2N)^4(I_2+I_4+2I_6)\left[\boldsymbol{\chi}(1,2)+\boldsymbol{\chi}(2,1)\right]$
W_{E5}	$\rightarrow$	$(g^2N)^4I_2\left[\boldsymbol{\chi}(1,2)+\boldsymbol{\chi}(2,1)\right]$
W_{E11}	$\rightarrow$	$(g^2N)^4(I_3+I_4+2I_5)\left[\boldsymbol{\chi}(1,2)+\boldsymbol{\chi}(2,1)\right]$
W_{E14}	$\rightarrow$	$-(g^2N)^4I_3\left[\boldsymbol{\chi}(1,2)+\boldsymbol{\chi}(2,1)\right]$
$\sum W_{E**}$	$\rightarrow$	$2(g^2N)^4(I_5-I_6)\left[\boldsymbol{\chi}(1,2)+\boldsymbol{\chi}(2,1)\right]$
W_{G11}	$\rightarrow$	$2(g^2N)^4I_1\,\boldsymbol{\chi}(1)$
W_{G29}	$\rightarrow$	$-2(g^2N)^4I_2\,\boldsymbol{\chi}(1)$
$\sum W_{G**}$	$\rightarrow$	$2(g^2N)^4(I_1-I_2)\,\boldsymbol{\chi}(1)$

Table 2: Four-loop wrapping contributions for the single impurity case.

Its contribution is given by

$$\begin{aligned}
S_4 &\rightarrow (g^2N)^4I_1\left[\boldsymbol{\chi}(1,2,3,4)+\boldsymbol{\chi}(4,3,2,1)\right] \\
&\sim \frac{5}{4}\frac{\lambda^4}{\varepsilon}\left[(\bar{q}^8+q^8)-2(\bar{q}^6+q^6)+(\bar{q}^4+q^4)\right] \\
&= \frac{5}{2}\frac{\lambda^4}{\varepsilon}\left[\cos(8\pi\beta)-2\cos(6\pi\beta)+\cos(4\pi\beta)\right]\ .
\end{aligned} \tag{4.2}$$

Thus the term to be subtracted from (4.1) is

$$\delta\gamma_4^{\rm s} = -8\lim_{\varepsilon\to0}(\varepsilon S_4) = -20\lambda^4\left[\cos(8\pi\beta)-2\cos(6\pi\beta)+\cos(4\pi\beta)\right]\ . \tag{4.3}$$

The relevant wrapping diagrams are given by a subset of those contributing to the two impurity case which are listed in figures 2 and C.1 to C.7 of [22]. In particular we need consider the diagram W_{A2} and all the ones belonging to the classes W_{B**}, W_{E**} and W_{G**}. The total contribution from each class can be read from table 2 with the momentum integrals given in table 3.

$$I_0 = \quad\text{(diagram)}\quad = \frac{1}{(4\pi)^6}\left(\frac{1}{6\varepsilon^3} - \frac{1}{2\varepsilon^2} + \frac{2}{3\varepsilon}\right)$$

$$I_1 = \quad\text{(diagram)}\quad = \frac{1}{(4\pi)^8}\left(-\frac{1}{24\varepsilon^4} + \frac{1}{4\varepsilon^3} - \frac{19}{24\varepsilon^2} + \frac{5}{4\varepsilon}\right)$$

$$I_2 = \quad\text{(diagram)}\quad = \frac{1}{(4\pi)^8}\left(-\frac{1}{24\varepsilon^4} + \frac{1}{4\varepsilon^3} - \frac{19}{24\varepsilon^2} + \frac{1}{\varepsilon}\left(\frac{5}{4} - \zeta(3)\right)\right)$$

$$I_3 = \quad\text{(diagram)}\quad = \frac{1}{(4\pi)^8}\left(-\frac{1}{12\varepsilon^4} + \frac{1}{3\varepsilon^3} - \frac{5}{12\varepsilon^2} - \frac{1}{\varepsilon}\left(\frac{1}{2} - \zeta(3)\right)\right)$$

$$I_4 = \quad\text{(diagram)}\quad = \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}5\zeta(5)$$

$$I_5 = \quad\text{(diagram)}\quad = \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}(-\zeta(3))$$

$$I_6 = \quad\text{(diagram)}\quad = \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}\left(\frac{1}{2}\zeta(3) - \frac{5}{2}\zeta(5)\right)$$

Table 3: Momentum integrals.

Now we need the explicit expressions for the chiral structures $\boldsymbol{\chi}(\dots)$ on single-impurity states. For the diagram W_{A2} we can deform the corresponding wrapping structure $\chi(1,4,3,2) + \chi(4,1,2,3)$ described in [22] or we can simply obtain the explicit expression from the diagram. We find

$$\boldsymbol{\chi}(1,4,3,2) + \boldsymbol{\chi}(4,1,2,3) = \left[(\bar{q}^8 + q^8) - 2(\bar{q}^6 + q^6) + (\bar{q}^4 + q^4)\right] \ . \tag{4.4}$$

All the other structures can be derived directly from the diagrams, as for W_{A2}, or computed using the general rules (2.5) and the definition of the deformed basis operators (2.4). We find

$$W_{A2} \sim \left[\frac{5}{4} - \zeta(3)\right]\frac{\lambda^4}{\varepsilon}\left[(\bar{q}^8 + q^8) - 2(\bar{q}^6 + q^6) + (\bar{q}^4 + q^4)\right]$$
$$= \left[\frac{5}{4} - \zeta(3)\right]\frac{\lambda^4}{\varepsilon}2\left[\cos(8\pi\beta) - 2\cos(6\pi\beta) + \cos(4\pi\beta)\right] \ , \tag{4.5}$$

$$\sum W_{B**} \sim \left[3\zeta(3) - 5\zeta(5)\right]\frac{\lambda^4}{\varepsilon}\left[(\bar{q}^6 + q^6) - 2(\bar{q}^4 + q^4) + (\bar{q}^2 + q^2)\right]$$
$$= \left[3\zeta(3) - 5\zeta(5)\right]\frac{\lambda^4}{\varepsilon}2\left[\cos(6\pi\beta) - 2\cos(4\pi\beta) + \cos(2\pi\beta)\right] \ , \tag{4.6}$$

$$\sum W_{E**} \sim [-3\zeta(3) + 5\zeta(5)] \frac{\lambda^4}{\varepsilon} \left[(\bar{q}^4 + q^4) - 2(\bar{q}^2 + q^2) + 2 \right]$$

$$= 2 [-3\zeta(3) + 5\zeta(5)] \frac{\lambda^4}{\varepsilon} [\cos(4\pi\beta) - 2\cos(2\pi\beta) + 1] , \quad (4.7)$$

$$\sum W_{G**} \sim 2\zeta(3) \frac{\lambda^4}{\varepsilon} \left[(\bar{q}^2 + q^2) - 2 \right]$$

$$= 4\zeta(3) \frac{\lambda^4}{\varepsilon} [\cos(2\pi\beta) - 1] . \quad (4.8)$$

Therefore the wrapping contribution to the anomalous dimension is given by

$$\delta\gamma_4^{\mathrm{w}} = -8 \lim_{\varepsilon \to 0} \left[\varepsilon \left(W_{A2} + \sum W_{B**} + \sum W_{E**} + \sum W_{G**} \right) \right] . \quad (4.9)$$

Finally collecting all the contributions we can write the exact anomalous dimension of $\mathcal{O}_{1,4}$ at four loops

$$\gamma_4(\mathcal{O}_{1,4}) = \gamma_4(\mathcal{O}_{\mathrm{as}}) - \delta\gamma_4^{\mathrm{s}} + \delta\gamma_4^{\mathrm{w}}$$

$$= -16\lambda^4 \big[160\sin^8(\pi\beta) - \zeta(3)\cos(8\pi\beta) + 5\big(\zeta(3) - \zeta(5)\big)\cos(6\pi\beta)$$

$$- \big(10\zeta(3) - 15\zeta(5)\big)\cos(4\pi\beta) + \big(11\zeta(3) - 15\zeta(5)\big)\cos(2\pi\beta)$$

$$- 5\big(\zeta(3) - \zeta(5)\big) \big] . \quad (4.10)$$

4.2 Two-impurity states

In this subsection we consider the length-four, two-impurity operators

$$\mathrm{tr}(\phi\phi ZZ) , \qquad \mathrm{tr}(\phi Z\phi Z) , \quad (4.11)$$

and our aim is to compute their anomalous dimensions at four loops.

We make use of the dilatation operator including wrapping interactions that we determined in [22], by adapting it to the deformed case through the substitution of the undeformed chiral structures with the ones given in (2.5)

$$\boldsymbol{D}_4^{\mathrm{sub}} + \boldsymbol{\delta D}_4^{\mathrm{w}} = (200 - 16\zeta(3))\boldsymbol{\chi}(1) - (150 - 24\zeta(3) + 40\zeta(5))[\boldsymbol{\chi}(1,2) + \boldsymbol{\chi}(2,1)]$$

$$+ (88 + 8\epsilon_{3a} + 24\zeta(3) - 40\zeta(5))\boldsymbol{\chi}(1,3)$$

$$+ (60 - 24\zeta(3) + 40\zeta(5))[\boldsymbol{\chi}(1,2,3) + \boldsymbol{\chi}(3,2,1)]$$

$$- \left(\frac{4}{3} - 8\zeta(3) - 4\epsilon_{3a} + 4i\epsilon_{3b} - 2i\epsilon_{3c} + 4i\epsilon_{3d} \right)\boldsymbol{\chi}(1,3,2)$$

$$- \left(\frac{20}{3} - 8\zeta(3) - 4\epsilon_{3a} - 4i\epsilon_{3b} + 2i\epsilon_{3c} - 4i\epsilon_{3d} \right)\boldsymbol{\chi}(2,1,3)$$

$$+ 4(1 - \zeta(3))\boldsymbol{\chi}(2,4,1,3) - (10 - 8\zeta(3))[\boldsymbol{\chi}(1,4,3,2) + \boldsymbol{\chi}(4,1,2,3)]$$

$$- (12 + 8\zeta(3) + 4\epsilon_{3a})\boldsymbol{\chi}(2,1,3,2) + (4 - 8\zeta(3))\boldsymbol{\chi}(4,1,3,2) , \quad (4.12)$$

where $\boldsymbol{D}_4^{\mathrm{sub}}$ contains the interactions with range up to four, while $\boldsymbol{\delta D}_4^{\mathrm{w}}$ contains the wrapping contributions [22].

Let us first notice that applying this operator to the single-impurity state $\mathcal{O}_{1,4}$ we immediately recover the four-loop anomalous dimension explicitly computed in the previous subsection and given in (4.10).

In order to compute the anomalous dimensions of the two-impurity operators (4.11) we have to consider the full dilatation operator up to four loops:

$$D = D_0 + \lambda D_1 + \lambda^2 D_2 + \lambda^3 D_3 + \lambda^4 \left(D_4^{\mathbf{sub}} + \delta D_4^{\mathbf{w}} \right) \, . \tag{4.13}$$

The application of this operator on the states (4.11) produces a mixing 2×2 matrix whose eigenvalues are the anomalous dimensions we are looking for. We can write them as

$$\gamma^{(\pm)} = 4 + \lambda \gamma_1^{(\pm)} + \lambda^2 \gamma_2^{(\pm)} + \lambda^3 \gamma_3^{(\pm)} + \lambda^4 \gamma_4^{(\pm)} \, . \tag{4.14}$$

Finally introducing the definition

$$\Delta(\beta) = \frac{\sqrt{5 + 4\cos(4\pi\beta)}}{3} \, , \tag{4.15}$$

we obtain the following results[5]

$$
\begin{aligned}
\gamma_1^{(\pm)} &= 6(1 \mp \Delta(\beta)) \, , \\
\gamma_2^{(\pm)} &= -3\big(5 + 3\Delta(\beta)^2\big) \pm \frac{3}{\Delta(\beta)}\big(1 + 7\Delta(\beta)^2\big) \, , \\
\gamma_3^{(\pm)} &= 6\big(19 + 9\Delta(\beta)^2\big) \pm \frac{3}{4\Delta(\beta)^3}\big(1 - 51\Delta(\beta)^2 - 165\Delta(\beta)^4 - 9\Delta(\beta)^6\big) \, , \\
\gamma_4^{(\pm)} &= -3\big(410 - 99\zeta(3) + 120\zeta(5)\big) - 18\Delta(\beta)^2\big(10 - 13\zeta(3) + 20\zeta(5)\big) \\
&\quad + 81\Delta(\beta)^4\big(2 - 3\zeta(3)\big) \\
&\quad \pm \frac{3}{8\Delta(\beta)^5}\big[1 - 44\Delta(\beta)^2 + 6\Delta(\beta)^4\big(189 + 4\zeta(3)\big) \\
&\quad\quad + 4\Delta(\beta)^6\big(809 - 468\zeta(3) + 480\zeta(5)\big) - 27\Delta(\beta)^8\big(37 - 40\zeta(3)\big)\big] \, ,
\end{aligned}
\tag{4.16}
$$

where the eigenstate with eigenvalue $\gamma^{(+)}$ becomes protected in the undeformed theory with $\Delta(\beta = 0) = 1$.

We notice that in the deformed theory no BPS state survives in the SU(2) sector. Thus, unlike in $\mathcal{N} = 4$, the eigenstates of the dilatation operator change with the loop order.

5. One-impurity states at higher orders

Now we study once again the simplest one-impurity operators and attempt to go beyond four loops. More specifically we look at the operator $\mathcal{O}_{1,L} = \text{tr}(\phi Z^{L-1})$ and analyze its anomalous dimension at higher order L in perturbation theory. Following our general strategy one would have to consider the asymptotic contributions from D_L. Then one would have to compute $\delta D_L^{\mathbf{s}}$ in order to subtract the range $L+1$ interactions. Finally one would need all the wrapping contributions $\delta D_L^{\mathbf{w}}$.

The first step is actually simple since we do not need the knowledge of the asymptotic D_L: the asymptotic contribution to the anomalous dimension of the single-impurity operator can be obtained directly from (2.8).

---

[5]After the appearance of [26] we corrected the rational part of this result.

Next we have to subtract the range $L + 1$ interactions. As discussed in the previous sections there is only one diagram to be subtracted, i.e. the range $(L + 1)$ graph denoted by S_L with the chiral structure $\chi(1, 2, \ldots, L)$.

Then we have to consider the wrapping diagrams. At L loops there will be wrapping contributions from one single diagram $W_{L,0}$ with only chiral interactions and from the classes with chiral structures

$$
\begin{aligned}
W_{L,L-1}: & \quad \chi(1) \;+\; (L-1) \text{ vectors}, \\
W_{L,L-2}: & \quad \chi(1,2) \;+\; (L-2) \text{ vectors}, \\
& \qquad\qquad \vdots \\
W_{L,1}: & \quad \chi(1,2,\ldots,L-1) \;+\; 1 \text{ vector}.
\end{aligned}
\tag{5.1}
$$

The general form of these contributions after D-algebra can be easily found for the structures with two, one and no vectors, for $L \geq 4$. At four loops we have the complete result

- $L = 4$:

$$
\begin{aligned}
W_{4,0} - S_4 &= C_{4,0}\frac{1}{\varepsilon}\zeta(3)\,, \\
W_{4,1} &= C_{4,1}\frac{1}{\varepsilon}[3\zeta(3) - 5\zeta(5)]\,, \\
W_{4,2} &= C_{4,2}\frac{1}{\varepsilon}[3\zeta(3) - 5\zeta(5)]\,, \\
W_{4,3} &= C_{4,3}\frac{1}{\varepsilon}\zeta(3)\,,
\end{aligned}
\tag{5.2}
$$

where the $C_{L,i}$ are rational prefactors.

Already at five loops wrapping diagrams with three vectors proliferate considerably and we have not embarked in their computations. We computed the momentum integrals corresponding to the classes with two, one and no vectors for $L = 5$ and 6, and the integrals for the cases of one and no vectors for $L = 7$. The results read

- $L = 5$:

$$
\begin{aligned}
W_{5,0} - S_5 &= C_{5,0}\frac{1}{\varepsilon}\zeta(5)\,, \\
W_{5,1} &= C_{5,1}\frac{1}{\varepsilon}[4\zeta(5) - 7\zeta(7)]\,, \\
W_{5,2} &= 0\,,
\end{aligned}
\tag{5.3}
$$

- $L = 6$:

$$
\begin{aligned}
W_{6,0} - S_6 &= C_{6,0}\frac{1}{\varepsilon}[4\zeta(5) + 35\zeta(7)]\,, \\
W_{6,1} &= C_{6,1}\frac{1}{\varepsilon}[20\zeta(5) + 49\zeta(7) - 126\zeta(9)]\,, \\
W_{6,2} &= C_{6,2}\frac{1}{\varepsilon}[10\zeta(5) - 7\zeta(7)]\,,
\end{aligned}
\tag{5.4}
$$

- $L = 7$:

$$W_{7,0} - S_7 = C_{7,0}\frac{1}{\varepsilon}[\zeta(7) + 6\zeta(9)]\,,$$

$$W_{7,1} = C_{7,1}\frac{1}{\varepsilon}\left[2\zeta(7) + 4\zeta(9) - 11\zeta(11)\right]\,. \tag{5.5}$$

Even with the partial results we have listed above, several comments are in order.

First of all, as a general observation, we recall that the anomalous dimensions of single-impurity states are not affected by the presence of a dressing phase. Therefore the transcendentality that we read in the results at the various loop orders is to be ascribed completely to finite size effects. We can summarize our findings as follows:

We have analyzed the anomalous dimensions of single-impurity states $\mathcal{O}_{1,L} = \mathrm{tr}(\phi Z^{L-1})$ at critical order, i.e. with operators with length equal to the loop order.

At order $L = 3$ we have found that subtraction and wrapping contributions cancel each other and the net contribution to the anomalous dimension is the same as from its asymptotic value. Wrapping at three loops seems to be irrelevant.

At order $L = 4$ we have computed exactly all the wrapping contributions to the anomalous dimension of $\mathcal{O}_{1,4}$ and found that the result contains terms proportional to $\zeta(3)$ and $\zeta(5)$.

Beyond four loops we have only partial results but a clear pattern seems to emerge: at every loop order the level of transcendentality is increased by two as compared to the previous order and no new rational part arises.

It becomes natural to compare this behavior to the one of the dressing phase: at three loop nothing happens, at four loop a contribution proportional to $\zeta(3)$ arises, at five loops $\zeta(5)$ appears and so on. What we have found indicates that finite size effects resemble the behavior of the dressing phase contributions at the various loop orders [41, 10], increasing by two the level of transcendentality.

Our calculation of the anomalous dimensions for the two-impurity state, where both the dressing phase and the wrapping contribute, confirms the above interpretation.

We hope that these results might shed some light on how to implement the wrapping interactions into a modified Bethe ansatz.

Several other issues are still completely open: among them we mention the fact that it would be important to compute finite size effects beyond critical order, i.e. compute the anomalous dimension of $\mathcal{O}_{1,L}$ at order $L+1$. Needless to say that now the major challenge resides in the comparison of the finite size contributions we have found in the weak coupling regime with the corresponding strong limit results from string theory computations.

Acknowledgments

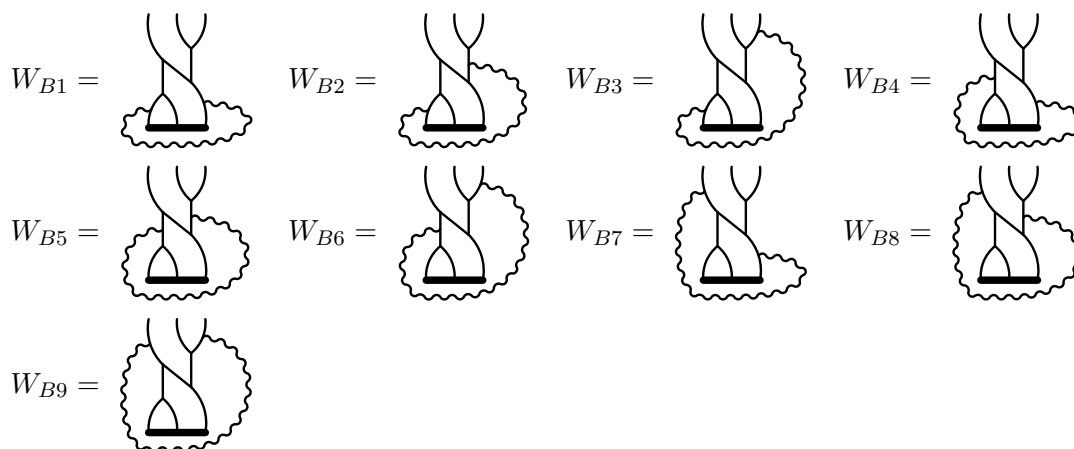## A. Cancellations for wrapping diagrams with vectors at three loops

Figure 3: Wrapping diagrams with chiral structure $\chi(2,1)$.

$W_{B1} \rightarrow$	$-W_{B2}$	$W_{B4} \rightarrow$	$-W_{B5}$	$W_{B7} \rightarrow$	$-W_{B8}$
$W_{B2} \rightarrow$	$-W_{B1}$	$W_{B5} \rightarrow$	$-W_{B4}$	$W_{B8} \rightarrow$	$-W_{B7}$
$W_{B3} \rightarrow$	finite	$W_{B6} \rightarrow$	finite	$W_{B9} \rightarrow$	finite

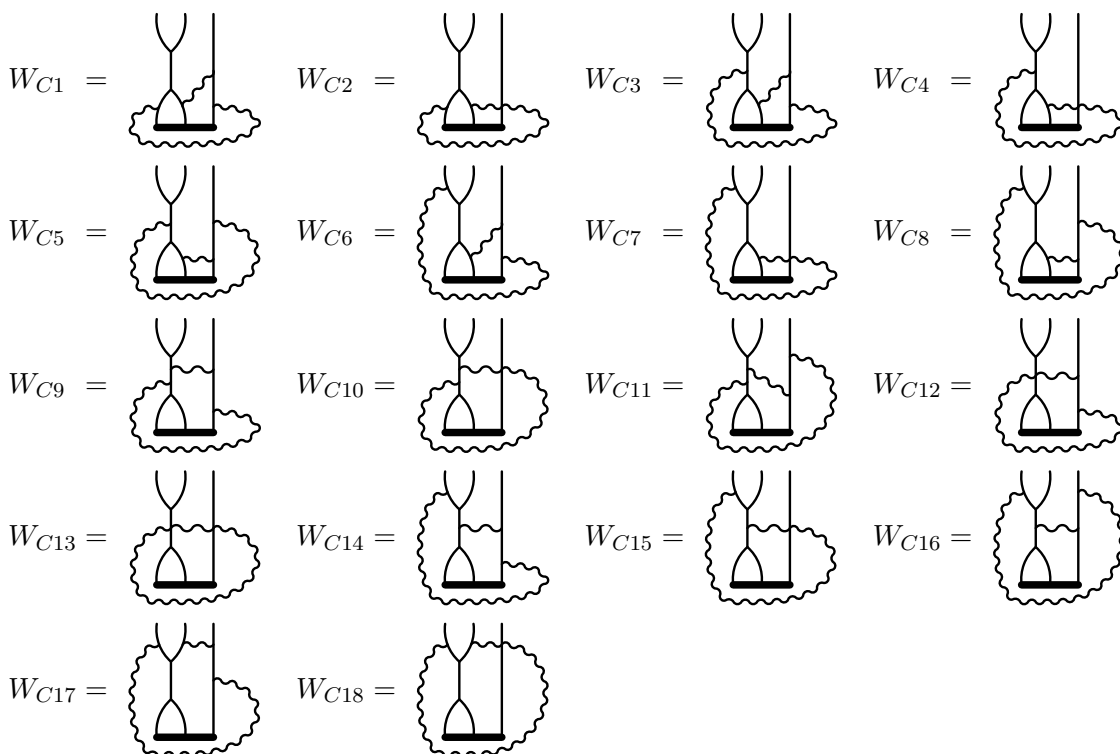**Table 4:** Cancellations for diagrams with structure $\chi(2,1)$.

W_{C1} = W_{C2} = W_{C3} = W_{C4} =

W_{C5} = W_{C6} = W_{C7} = W_{C8} =

W_{C9} = W_{C10} = W_{C11} = W_{C12} =

W_{C13} = W_{C14} = W_{C15} = W_{C16} =

W_{C17} = W_{C18} =

Figure 4: Wrapping diagrams with chiral structure $\boldsymbol{\chi}(1)$.

W_{C1}	$\to$	$-W_{C5}$	W_{C6}	$\to$	finite	W_{C11}	$\to$	$-W_{C3}$	W_{C16}	$\to$	finite
W_{C2}	$\to$	finite	W_{C7}	$\to$	finite	W_{C12}	$\to$	$-W_{C9}$	W_{C17}	$\to$	finite
W_{C3}	$\to$	$-W_{C11}$	W_{C8}	$\to$	finite	W_{C13}	$\to$	finite	W_{C18}	$\to$	finite
W_{C4}	$\to$	$-W_{C10}$	W_{C9}	$\to$	$-W_{C12}$	W_{C14}	$\to$	finite			
W_{C5}	$\to$	$-W_{C1}$	W_{C10}	$\to$	$-W_{C4}$	W_{C15}	$\to$	finite			

Table 5: Cancellations for diagrams with structure $\boldsymbol{\chi}(1)$.

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